An Application Based Approach to Teach Mathematics to Undergraduate Students Especially Engineers

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Abstract
It is a common belief among engineering educators that the engineering undergraduates could be more effectively prepared in mathematics when taking courses related to their respective engineering fields. Engineering being a practitioner’s discipline requires creative application of mathematical concepts to solve engineering problems. However, engineering students taking courses in mathematics are seldom exposed to a broad spectrum of engineering applications. This manuscript focuses on the development of an instructional approach to effectively teach mathematics to undergraduate engineering students. Integration of engineering applications into mathematics courses can enhance the conceptual understanding of mathematics. This instructional approach is in line with the increasing emphasis of Application-Based Engineering teaching on the design and computational abilities of undergraduate engineering students. The aforementioned instructional approach helps in understanding the mathematical concepts and their engineering applications. This approach enhances students’ interest in mathematics; it becomes clear to the engineering students that mathematics is not just the manipulation of formulae but is a useful tool applied to different physical phenomenon. Understanding the logic behind the formula will convince the students that engineering and mathematics are interconnected. Examples are provided in the manuscript to explain how this instructional strategy aids engineering students in understanding the use of mathematics in solving engineering problems.

Keywords: Critical thinking, Survey, Applications

Introduction
In most of the engineering programs around the globe, the required mathematical tools are taught separately from the engineering point of view and the students are rarely shown the applications of the mathematical courses. This isolation of mathematics and engineering has led to a pre-conceived notion amongst the students that mathematics is useless from the perspective of engineering.

Sometimes elegance explanations offered in the classroom are too brief to clear the mathematical concepts. This gives an undeserved sense of intellect to students. As long as a student is getting good grades, he/she does not feel anything wrong but when he/she enters into practical life then he/she realizes what he/she has missed.

In many universities the courses offered by the mathematics departments are not just for the engineers specifically but also cater for the students of economics, social science and other disciplines. When the material taught in such courses is not application-based then the students (from other disciplines), who may have sound mathematical skills, are unable to analyze real-life phenomenon mathematically.

As engineering has become an increasingly technology oriented profession, therefore a solid background in application-based mathematics is needed. Bearing this in mind if an effort is made to teach students how mathematics is interlinked with the engineering by explaining the mathematical concepts and their practical applications, it is likely that the engineering students will be motivated to study mathematics.

Sohail Anwar and Shamsa Anwar [3] described the usefulness of the integrated learning modules to teach engineering technology students. They conducted the students’ survey showing popularity of these modules; Klingbeil, Mercer, Rattan, Raymer, and Reynolds [2] worked on a model to increase student retention, and success in engineering through Mathematics instruction which proved to be very successful as well.

Application Based Teaching approach enhances students' understanding regarding the usefulness of mathematics in real-world engineering problems. If this approach becomes part of the curriculum then it develops active learning by engaging students in higher-order thinking related to real-life situations.

Development of Application-Based Teaching
In every technology everybody needs to have strong abilities to understand and implement the results in mathematical form. Application-based teaching helps in understanding the mathematical proofs and to relate the formulae with the physical applications.

Consider the following comments of a student taken from a course evaluation:

“Vector Projections were being covered in Calculus II and Linear Algebra simultaneously and I happened to have both courses. In Calculus II several questions were done in the class but none of the students manage to follow. The same thing was covered in Linear Algebra and instructor spent a fraction of the class on the topic and majority of the students quickly absorbed the basic idea.”
He connected the concept with a practical phenomenon by saying if an Aero-plane takes off then considers its path as the resultant vector and the path followed by its shadow on the ground as the horizontal projection of the resultant vector.

In the above comment a very easy example is given to explain projections practically but it is a common experience that the simple topics are taught without creating any insight among students for example curl of a vector, and divergence of a vector are taught in such a way so that students know that curl is only a cross product and divergence is a dot product in which one vector is ordinary and other one is in differential form.

Instructor must spent time in proving that curl is related to rotation and if curl of a vector field is zero then field is irrotational otherwise curl would point upward or downward provided rotation is counterclockwise or clockwise. This could be further related to an example of a Tornado. Similarly divergence could be made more interesting by explaining that if it is negative at a point then that point is a sink (where fluid vanishes) and if positive then it is a source (where fluid emerges).

In addition it is also observed that by using colorful slides/multimedia one can draw diagrams (difficult to do on board) by which explaining the concepts becomes easier. For example, in FIGURE NO. 1, it is explained that there are 8 Octants in 3 dimensions (which is bit complicated in comparison with the concept of 4 quadrants in 2 dimensions). Making colorful presentations involves a lot of efforts but students understand and concentrate very well.

Even if we take a difficult topic for example TENSORS, then by using colorful slides one can motivate students that TENSORS are introduced for simplification but not to make mathematics difficult.

By relating to a physical phenomenon for example stresses acting on three faces of a rectangular box in first octant one can easily explain that instead of introducing three vectors we can construct one tensor with matrix representation given by

\[
\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix}
\]

which gives all the 9 forces (components of a tensor), \( \sigma_{ij}, i, j \in \{1,2,3\} \), where i indicates the face with normal Vector \( e_i \), and j is indicating the direction. SEE FIGURE 2

Briefly it is most important to relate the mathematical concepts with daily life situations in order to prove the significance of mathematics.

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**Fig 1:** Three co-ordinate planes divide space into EIGHT OCTANTS.
Fig 2: 9 components of the stress tensor acting in the positive directions of the coordinate axes

Use of Method

A survey questionnaire (TABLE A1) was conducted in the beginning of the semester to assess the effectiveness of application-based teaching. There are mathematics courses, which are really important for the engineering students e.g. Multivariate Calculus, Complex Analysis, Linear Algebra and Differential equations. But in addition to the engineering majors, students from other disciplines take these courses as well. This survey is taken not only from the engineering students but students from other disciplines e.g. Economics etc. as well, but majority of the senior engineering students enrolled responded to the survey. A total of forty-eight students filled out the questionnaire. Table A2 shows the number and the percentage of respondents corresponding to each of the rating for the questionnaire items except the last item in which the surveyors had to give their response regarding the topics they had found useful.

<table>
<thead>
<tr>
<th>Table A1: Questionnaire for the assessment of the Application based teaching modules.</th>
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<tbody>
<tr>
<td>Ratings used are as follows:</td>
</tr>
<tr>
<td>A = strongly disagree</td>
</tr>
<tr>
<td>B = disagree</td>
</tr>
<tr>
<td>C = neutral</td>
</tr>
<tr>
<td>D = agree</td>
</tr>
<tr>
<td>E = strongly agree</td>
</tr>
</tbody>
</table>

To assess the application based teaching methods; following questions were asked from the Senior Level students after completing the stream of Mathematics courses:

1. Applications of the topics into daily life were discussed
2. Use of Multimedia/color slides improved the learning process
3. Weekly quizzes and assignments helped in understanding the course material
4. Problem solving skills were developed
5. Understanding the logic (proofs) of the formulae developed the insight into different types of applications
6. Application based mathematics courses were helpful in making the concepts clearer in comparison with the non-application based.
7. The pace of the courses was appropriate (neither too fast nor too slow)
8. Class strength must be reasonable so that the interaction with the students make the atmosphere more conducive to learning

9. The class time (early morning/afternoon/late evening) affected the learning process

10. Without Mathematical insight reasonable progress in the engineering/anticipated career is difficult

11. Which topics did you find useful for your future studies? Please write down a few sentences.

Table A2: Number and percentage of respondents corresponding to each of the ten items in the survey

<table>
<thead>
<tr>
<th>Questionnaire item</th>
<th>Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applications of the topics into daily life were discussed</td>
<td>A (0%)</td>
</tr>
<tr>
<td>Use of Multimedia/color slides improved the learning process</td>
<td>A (6%)</td>
</tr>
<tr>
<td>Weekly un-announced quizzes and assignments helped in understanding the course material</td>
<td>A (0%)</td>
</tr>
<tr>
<td>Problem solving skills were developed</td>
<td>A (0%)</td>
</tr>
<tr>
<td>Understanding the logic (proofs) of the formulae developed the insight towards different types of applications</td>
<td>A (2%)</td>
</tr>
<tr>
<td>Application based mathematics courses were helpful in making the concepts clearer in comparison with the non-application based.</td>
<td>A (0%)</td>
</tr>
<tr>
<td>The pace of the courses was appropriate (neither too fast nor too slow)</td>
<td>A (0%)</td>
</tr>
<tr>
<td>Class strength must be reasonable so that the interaction with the students makes the atmosphere more conducive to learning</td>
<td>A (0%)</td>
</tr>
<tr>
<td>The class time (early morning/afternoon/late evening) affected the learning process</td>
<td>A (38%)</td>
</tr>
<tr>
<td>Without mathematical insight reasonable progress in the engineering/anticipated career is difficult</td>
<td>YES</td>
</tr>
</tbody>
</table>

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Questionnaire item 11
Which mathematics topics did you find useful for your future studies? Please write down a few sentences. More explanation and selected students' responses (Table A3) are given after the analysis of this table's statistics.

Table A2 shows that a total of 79% of the students either agreed or strongly agreed with the statement that the applications of the topics into daily life were discussed. About 15% of the students had a neutral response to this statement and about 6% disagreed.

A total of 94% of the students either agreed or strongly agreed with the statement that the use of Multimedia/color slides improved the learning process. About 6% of the students strongly disagreed with this statement.

A total of 87% of the students either agreed or strongly agreed with the statement that the weekly quizzes and assignments helped them in understanding the course material. About 11% of the students had a neutral response to this statement and about 2% disagreed.

A total of 83% of the students agreed or strongly agreed with the statement that their problem solving skills were developed. About 15% of the students had a neutral response to this statement and about 2% disagreed.

A total of 90% of the students either agreed or strongly agreed with the statement that the weekly quizzes and assignments helped them in understanding the course material. About 8% of the students had a neutral response to this statement.

A total of 84% of the students either agreed or strongly agreed with the statement that the application based mathematics courses were helpful in making the concepts clearer in comparison with the non-application based. About 8% of the students had a neutral response to this statement and about 8% disagreed.

A total of 58% of the students either agreed or strongly agreed with the statement that the pace of the courses was appropriate (neither too fast nor too slow). About 30% of the students had a neutral response to this statement and about 12% disagreed.

A total of 63% of the students either agreed or strongly agreed with the statement that the class strength must be reasonable so that the interaction with the students makes the atmosphere more conducive to learning. About 21% of the students had a neutral response to this statement and about 16% disagreed.

A total of 61% students either disagreed or strongly disagreed with the statement that the class time (early morning/afternoon/late evening) affected the learning process. About 21% had a neutral response to this statement. About 18% of the students either agreed or strongly agreed with this statement.

A total of 98% of the students had a cumulative response to the statement that without mathematical insight reasonable progress in the engineering/anticipated career is difficult and about 2% disagreed.

Item 11 is further to item 10 and designed to know whether students study mathematics courses for fulfilling their degree requirements or are they convinced that mathematics is interconnected with the other disciplines? For example the theory of Cartesian Tensors is important to solve Non-Linear Elasticity problems.

Differential equations play an important role to tackle courses like network analysis etc. But this is only possible if instructor teaches Cartesian Tensors and applies this theory to Elasticity problems in the same course and adopts the similar application-based techniques while teaching other mathematics courses.

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>Selected Students' Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cartesian Tensors is an important and interesting topic for understanding the practical theory of Continuum Mechanics, and Computational Fluid Dynamics.</td>
</tr>
<tr>
<td>2</td>
<td>Generalized Tensors and Inner product spaces would be helpful when studying Relativity, and Quantum Physics.</td>
</tr>
<tr>
<td>3</td>
<td>One cannot even think of entering into the Engineering without a solid background in Applied Mathematics especially Calculus problem solving techniques.</td>
</tr>
<tr>
<td>4</td>
<td>Linear Algebra concepts are used in Computer Graphics, like rotation matrices. Courses like Analog Digital Communication, neural networks are impossible without strong mathematical background.</td>
</tr>
<tr>
<td>5</td>
<td>Matrices, Complex Numbers, Vectors, Trigonometric functions, Contour Integration, Euler’s formula, Eigen vectors, and Differential Equations are extensively used in circuit solving, electronics, signal and systems, Data Mining Algorithms and Wavelet Transformations.</td>
</tr>
</tbody>
</table>

Table A3: containing responses from the students regarding item 11.
Conclusions

This paper describes the use of Application-based teaching of Mathematics, which illustrates an instructional approach for developing insight into mathematical rules and how much mathematics plays role into real life.

Use of multimedia and colorful slides makes easier to grasp concepts, shows dedication and efforts to ensure that every student is following the class. In addition at undergraduate level students must be encouraged towards proof writing and to solve exams without the unnecessary aids like open book/notes.

Not doing the proofs and using open book/notes methods in exams leads to accept mathematical results without any logic and insight. They might solve a lot of questions for example on Integration but no idea what Integration is according to geometrical point of view and what are its applications. Application-based teaching proves that Mathematics is not just about learning the formulas (as some people believe) but understanding the concepts and applications.

Students realize that Mathematics can be taught without learning concepts by heart and leads to a better understanding of physical phenomenon. This gives a good knowledge of things, which students assumed to be true. This results in maximum learning in the class (criteria for a good teacher).

Students felt that how much instructor is concerned about student’s development and value-addition. It explains difficult and abstract topics with such adeptness that they would become crystal clear. It improves instructor’s ability to know how to pick up a new topic in class and to make the flow of thoughts connected to some physical situation in daily life and make it understandable. The problem lies in the strategy of instructing. If a student is shown the practical implementation of a theory right from the class, student may think out of the classroom scenario and will relate the formulae of the text with real life objects. Instructor should discuss how the theory relates with real time inventions and how it was intercepted previously.

Learning the concept is more important than getting the answers right. Explaining the theory, deriving the expression, this ensures that students remember the stuff and understand it practically. An application based teaching course encourages students to step towards real learning and discourages the culture of grade-consciousness, rote learning and false practices.

Mathematics plays an important role as a computational tool applicable in natural science, social science and engineering problems. For example see references [4, 5, 6, 7]. Giving examples from daily life makes different topics interesting to understand mathematics like [1].

References